Simple but very useful models of the atmosphere

3.1) Basic descriptions of the atmosphere

3.1.1) Vertical atmospheric processes

The vertical is the second most important dimension, after latitude, because there are...
Strong gradients (of temperature, humidity, etc)
Significant flux divergences
(N.B. the zonal *fluxes* are also large, but their divergences are quite small)
The most important vertical processes are
convection
large-scale ascent & descent (subsidence) i.e. the meridional circulation
small-scale turbulent overturning & mixing (unstable stratification)
radiation absorption and re-emission (some SW, but especially LW)

3.1.2) Convection & Atmospheric lapse rates

The troposphere is (on average) just stable, but there are major differences between regions of ascent (active convection) and descent (subsidence).
In **ascending** regions (which are small) the stratification is slightly unstable, and the air is mostly saturated with water vapour (because of cooling and condensation). The lapse rates tend towards the moist adiabatic value, i.e. $\Gamma < 6$ °C/km.
In **descending** regions (which are large) the stratification is slightly stable, and the air is under-saturated (because it has been dried out by condensation & precipitation during its ascent). The lapse rate tends towards the dry adiabatic, i.e. $\Gamma \approx 10$ °C/km.
As a result, large-scale spatially averaged lapse rates are actually rather close to 6.5 °C/km almost everywhere (because of lateral mixing by eddies, etc)

3.1.3) Relative Humidity

The relative humidity of the air is likewise high (≈100%) in ascending air (where condensation occurs), and low (usually much less than 60%) in the descending dry air.

Over the sea, evaporation causes RH to increase rapidly (to ≈ 85% or more). Over land, the humidity depends on P-E (Precipitation-Evaporation), which is
high in the tropics (in the ITCZ) , and at mid-latitudes, near ± 60 ° N/S
low around ± 30 ° latitude and near the pole, because of subsidence, (which is why there are deserts there...)
Overall, relative humidity (at ground level) is in the range 75 ± 20 % almost everywhere (but NB “almost everywhere” is mostly over the sea !!)

3.1.4) The “US standard” atmosphere

A very basic description of the atmosphere is given by the “US standard” atmosphere, which has

- lapse rate = -6.5 °C/km everywhere
- RH = 75 % everywhere

Ascent (and excess precipitation) near the equator and around ± 60 ° latitude
Subsidence (and excess evaporation) near the poles and around ± 30 ° latitude
Thus the meridional circulation is broadly described in terms of the Hadley and Ferrel circulation cells.

3.2) Radiative-convective models of "grey" atmospheres

3.2.1) Introduction

We shall use the “grey” atmosphere model, i.e. where there is partial absorption of infra-red radiation, which **independent of wavelength.** Thus atmosphere has a finite optical thickness in the infra-red (so it is optically neither very thick nor very thin). The simplest case is to consider the two-stream approximation, in which there are vertical fluxes (up-welling and down-welling) of thermal infra-red radiation only. We seek to deduce the temperature gradient (and thus T_s) for various cases (e.g. levels of insolation, optical thickness, etc). The following treatment is based on [Salby, 1992 #3107](in [Trenberth, 1992 #3185]) and [Houghton, 1997 #3186]. See also [Goody, 1995 #486] for a more complete treatment.

Divide the atmosphere into thin layers, each of optical thickness $\Delta\tau$, and let τ be measured downwards from the top of the atmosphere (TOA), like pressure. Thus τ is a function of altitude and pressure (p), and for a dry atmosphere may be taken to be proportional to pressure. If the upward infra-red (IR) flux is F_{up}, and the downward IR flux is $F_{dn,}$ then the net IR flux $F_{net} = F_{up} - F_{dn}$. The net upward IR flux at the top of the atmosphere (TOA) is $F_0 = F_{net}(0)$, which by energy balance must moreover be equal to the net absorbed solar (SW) radiation, so $F_0 = (1 - \alpha)$ S.

Now, consider the augmentation and dimunition of both F_{up} and F_{dn} for each thin slab of atmosphere, of optical thickness $\Delta\tau$, due to the absorption and (black-body) re-radiation of the infra-red radiation (see Figure 1). Remembering that the emissivity of each layer is the same as its absorption coefficient (which is, by the definition of optical thickness, just equal to $\Delta\tau$), and considering first the downwelling radiation (which is oriented in the same direction as τ, i.e. downwards from the top of the atmosphere) one finds that

$$\Delta F_{dn} = - F_{dn}\,\Delta\tau + B(T)\,\Delta\tau, \tag{1}$$

where $B(T) = \sigma T^4$ (the usual expression for black – body radiation from a surface) so that any reduction of F_{dn} by absorption is opposed by the increment due to the re-radiation occurring *within* the thin slab. Note also that while B is fundamentally a function of local air temperature, it may also be regarded as a function of altitude, pressure or optical thickness, whichever is more convenient, since temperature is a function of all of these variables too. Taking the limit of infinitesimally thin layers, we have

$$dF_{dn}/d\tau = - F_{dn} + B(T) \tag{2}$$

Note that F_{dn} is zero at the top of the atmosphere, and increases downwards, because B(T) is always positive, and thus initially (and in fact always) exceeds $F_{dn.}$ The case of F_{up} is potentially (and actually) quite confusing, because F_{up} is orientated in the *opposite* direction to τ, so that the sign of ΔF_{up} across the slab is reversed, and thus

$$– \Delta F_{up} = – F_{up} \Delta\tau + B(T) \Delta\tau, \tag{3}$$

and so, again taking the infinitesimal limit

$$dF_{up}/d\tau = F_{up} – B(T). \tag{4}$$

These expressions for F_{up} and F_{dn} may be integrated directly (but usually this must be done numerically) for any given temperature profile T(z), provided that we know τ as a function of altitude i.e. $\tau(z)$ and vice versa. However, in general T(z) is not known *a priori* and must also be determined. We shall consider how to do this in the case of both pure radiative equilibrium, and for the radiative-convective case in which convection occurs if the temperature profile becomes unstably stratified. Before doing so it is useful to derive the expressions for the inter-relation between F_{tot} and F_{net}. Adding and subtracting the expressions (2) and (4) for F_{up} and F_{dn} we obtain

$$dF_{net}/d\tau = dF_{up}/d\tau \;–\; dF_{dn}/d\tau\ , \quad \therefore\ dF_{net}/d\tau \;=\; F_{tot} – 2B(T) \tag{5}$$

and

$$dF_{tot}/d\tau = dF_{up}/d\tau \;+\; dF_{dn}/d\tau \;=\; F_{up} – F_{dn}\ , \quad \text{i.e. } dF_{tot}/d\tau = \; F_{net} \tag{6}$$

Thus the rate of change (w.r.t. τ, and thus also altitude) of the total flux depends only on the net flux F_{net} (and is in fact proportional to it), and does not depend on temperature at all. The rate of change of the net flux, however, is in general determined by the imbalance between the average flux ($F_{tot}/2$) and the value of B(T).

3.2.2) Pure Radiative Equilibrium

We consider first the case in which the medium is static (i.e. there is no convection or small-scale mixing, etc), so that only the radiative processes are in operation. To derive the equilibrium conditions, we assume that we have local energy balance, which is thus (by assumption) due to radiative processes only. At thermal equilibrium, the divergence of the net IR flux must be zero, so $dF_{net}/dz = 0$, and thus also $dF_{net}/d\tau = 0$, and F_{net} must be a constant. This constant must thus also be just equal to the net upward IR flux at the top of the atmosphere, i.e.

$$F_{net} = F_0 \text{ , everywhere} \tag{7}$$

However, from equation (5) we also find that

$$dF_{net}/d\tau = F_{tot} - 2B(T) = 0$$

and in this case we therefore deduce that $F_{tot} = 2B(T)$. Thus, for this case of pure radiative equilibrium, the local temperature must be just such that

$$B(T) = F_{tot}/2. \tag{8}$$

However, recalling that $F_{net} = F_0$, which is a constant, we may also easily integrate the expression (6) for $dF_{tot}/d\tau$, as

$$F_{tot} = \int F_0 d\tau = F_0 \int d\tau = F_0\tau + \text{const} \tag{9}$$

Since F_{dn} is zero at the top of the atmosphere (where $\tau = 0$), $F_{tot}(0) = F_{up}(0) = F_0$, and the constant of integration must also be equal to F_0, and so

$$F_{tot} = F_0 (\tau + 1) \tag{10}$$

Now, in order to find the temperature as a function of τ, and thus of altitude, we shall consider B as a function of optical thickness $B(\tau)$ rather than of temperature. Recalling (from equation 8) that $B(T) = F_{tot}/2$, we may now write

$$B(\tau) = \sigma T^4 = F_0 (\tau + 1)/2 \tag{11}$$

From this expression we can now see that the temperature at the altitude where the optical thickness (depth) is 1 must be such that $[B]_{\tau=1} = B(1) = F_0 = \sigma T_{eff}^4$. This justifies the statement that the actual air temperature is equal to the effective radiative temperature, not at the top of the atmosphere, but at the altitude where $\tau = 1$. In fact, at the top of the atmosphere, where $\tau = 0$, we have $B(0) = F_0/2$, so that the air temperature tends to the (lower) value of

$$T_{TOA} = (F_0/2\sigma)^{1/4}$$

(12)

In order to determine the temperature at the ground surface (T_g) for this purely radiative equilibrium, we need to consider the upward flux of infra-red radiation, since

$$[F_{up}]_{z=0} = \sigma T_g^4$$

(13)

Since $F_{up} = (F_{tot} + F_{net})/2 = (F_{tot} + F_0)/2$, we find using equation (10) that

$$F_{up} = \{F_0 (\tau + 1) + F_0\}/2 = F_0 (1 + \tau/2)$$

(14)

Finally therefore, we also deduce that

$$F_{dn} = F_{up} - F_{net} = F_{up} - F_0 = F_0 (\tau/2)$$

(15)

Thus in the special case of pure radiative equilibrium, F_{net} is constant and equal to F_0, and both F_{up} and F_{dn} increase linearly with optical thickness. This is illustrated in Figure 2 (see also [Salby, 1992 #3107][Houghton, 1997 #3186]).

However, it is very important to notice that the (ground) surface temperature is set by F_{up} through equation (13), i.e.

$$\sigma T_g^4 = [F_{up}]_{z=0} = F_0 (1 + \tau/2)$$

(16)

whereas the air temperature just above the ground is set by F_{tot} through equation (11) so that

$$\sigma T_0^4 = B(\tau) = F_0 (\tau + 1)/2$$

(17)

Problems with the pure radiative model

There are several problems with the results which we have now obtained. Firstly, the ground surface temperature derived above exceeds that of the overlying air in this model, by an amount corresponding to an extra heat flux of $F_0/2$. This calculated ground-air temperature discontinuity may be substantial (10 or 20 °K, or more). It only occurs because we have assumed that the only heat fluxes are those due to radiation, so there is no conduction and no turbulent convection. In the real atmosphere these would operate together, as conduction will transfer heat into the air near the ground, creating an unstable stratification which will cause convection to occur. For this reason the pure radiative model is unsatisfactory for a real conductive, fluid atmosphere.

Other (related) problems can also be identified. From the results above, the effective IR emissivity (or transmission coefficient) of the atmosphere is

$$\varepsilon = F_0/F_{up}(\tau_g) = 1/(1 + \tau_g/2) \tag{18}$$

where τ_g is the optical thickness of the whole atmosphere, integrated from the top to the ground. Since we know from observations and the simple energy balance model that the effective value of ε is about 0.65, we can deduce that τ_g needs to be about 1, to be consistent with the observations. This is substantially less than the actual measured values of τ_g, which are about 4 at mid-latitudes.

More precisely, since $\sigma T_g^4 = F_0 (1 + \tau_g/2)$, and $F_0 = \sigma T_{eff}^4$, we have

$$T_g^4 = T_{eff}^4 (1 + \tau_g/2), \text{ and thus}$$

$$\tau_g = 2(T_g^4/T_{eff}^4 - 1) \tag{19}$$

Taking $T_g = 288$K, and $T_{eff} = 255$K, this implies that $\tau_g = 1.254$. However, we saw above that T_{eff} is (or should be) the temperature at the altitude H_{eff} where τ=1. Taking τ to be proportional to pressure, this implies that

$$H_{eff} = (1/1.254)\text{x } 1000 \text{ mbar} \approx 800 \text{ mbar} \tag{20}$$

However, the temperature difference $\Delta T = T_g - T_{eff} = 33$ K and using an atmospheric lapse rate of 6.5 °C/km, this implies that $H_{eff} = 5$ km which corresponds to a much greater altitude of about 500 mbar.

These problems indicate that the pure radiative model is inconsistent with several key observations, and is not an adequate first-order model for the heat transport processes of the lower atmosphere. This is not a big surprise. To fix the problems we need to consider firstly the effects of convection, and secondly the effect of the very non-uniform distribution of water vapour in the atmosphere.

3.2.3) Convective adjustment, and the "dry" radiative-convective model

In Figure 3 we plot the temperature profiles corresponding to equation (11) for values of τ_g in the range 1 to 10, along with a profile corresponding to the adiabatic lapse rate. We see that in the upper atmosphere the radiative profiles are always stably stratified, so the static radiative model should be a reasonable approximation (because conduction is trivial compared with turbulent convection). In the lower atmosphere, however, the temperature profiles are always unstable (super-adiabatic) if we take the discontinuity at the ground into account, and sometimes unstable even at mid-altitude for large values of τ_g .

**** Stratosphere/Troposphere

The simplest assumption to make would be that the temperature profile should be replaced by the adiabatic profile if it is unstable, but not otherwise. However, this leads to local air temperatures which are different from those implied by equation (11), so that the analytical treatment of section 3.2.2 (which implicitly assumes that they are the same) is no longer valid. We must therefore return to the more general equations of section 3.2.1, and integrate equations (5) and (6) using the actual air temperatures deduced from the adiabatic profile, which can in general only be done numerically (but see also [Pierrehumbert, 2002 #3187] for some interesting special cases).

Pseudo-code for a radiative-convective calculation

calculate radiative fluxes (divergence)

update temperature profile

if unstable w.r.t. chosen lapse rate

apply convective mixing → desired lapse rate

(conserve heat, water, etc)

→ implied convective heat flux...

repeat → radiative-convective equilibrium

→ tropo-pause & (unrealistic) stratosphere

1-D RCM's : features

Can include various radiatively active gases

(water vapour, ozone, CO_2, methane etc...)

better representation of stratosphere...

allows direct estimation of GH effects

and thus climate sensitivity

clouds : e.g. if RH > $RH_{crit} \approx 90$ %

specify albedo (≈ 0.5) or estimate (diffuse scattering)

specify cloud height & depth

fixed cloud top height, or temperature (?)

several cloud layers ? (how to model ?)

1-D RCM's : in practice

Consider

many (≈ 20) layers

many radiatively active "species" (gases etc)

integration over many spectral lines and bands, and over a continuum (8 to 13 μm)

both UV/Visible and IR radiation

particulate scattering...

Complex and time-consuming calculations...!

Computational demand of radiation code may exceed that of fluid flow, in GCM's

1-D Radiative-Convective Models

valid locally (isolated, pointwise), or global mean

but results vary with latitude/insolation

$\rightarrow$ latitudinal variation of tropo-pause height, etc

but $\Rightarrow$ inconsistency : not in local vertical balance

need to allow for lateral transports

$\Rightarrow$ need 2-D (meridional/vertical) model (at least !)