Simple Climate Models

Lecture 2

One-dimensional (meridional) Energy Balance Models

Meridional Non-uniformity

- ♦ Insolation varies with latitude
  - NB : need to allow for the effect of obliquity
- ♦ geometrical effects
  - projected and actual areas of latitude bands
- ♦ pole-equator temperature differences
  - moderated by atmospheric and oceanic transports of heat (and also water, etc...)
  - ∴ need to include advection & diffusion , i.e. transport by mean circulation and by eddies

1D (meridional) EBM's

- Incoming SW solar radiation
  - varies as a function of latitude
  - and of albedo (may be a function of temperature...)
- Outgoing LW infra-red radiation
  - as a function of local surface temperature
- Heat transport, by mixing (and maybe advection)
  - needs to be parameterised
- Heat capacity of land/sea surface (mixed layer?)
  - needed for time-dependent calculations only
- Excellent review by North et al (1981)
  - North GR, Cahalan RF & Coakley JA, Rev Geophys & Space Physics, 19, 91-121 (1981)

Geometrical details (1)

actual surface areas (for OLWR)

Within latitude bands $(\theta - \Delta\theta / 2)$ and $(\theta + \Delta\theta / 2)$...

(a) actual surface area

$$dA_s = 2\pi R \cos\theta \; R \, d\theta \quad = \quad 2\pi R^2 \cos\theta \, d\theta$$

$$\therefore A_s = \left[2\pi R^2 \sin\theta\right]_{\theta - \Delta\theta/2}^{\theta + \Delta\theta/2}$$

$$= 2\pi R^2 \{\sin(\theta + \Delta\theta / 2) - \sin(\theta - \Delta\theta / 2)\}$$

$\rightarrow 4\pi R^2$ for whole sphere

Geometrical details (2)

Projected surface areas (for ISWR)

(b) equatorial projected area

$$dA_p = 2R\cos q \; R\,dq \, \cos q \quad = \quad 2R^2 \cos^2 q \, dq$$

$$= R^2 (1 + \cos 2q)\, dq$$

$$\therefore \; A_p = R^2 \left[q + \frac{1}{2} \sin 2q \right]_{q - \Delta q/2}^{q + \Delta q/2}$$

$$= R^2 \left[\Delta q + \frac{1}{2} \left\{ \sin\left(2q + \Delta q\right) - \sin\left(2q - \Delta q\right) \right\} \right]$$

$$\rightarrow p \; R^2 / 2 \;\; \text{for a hemisphere : OK}$$

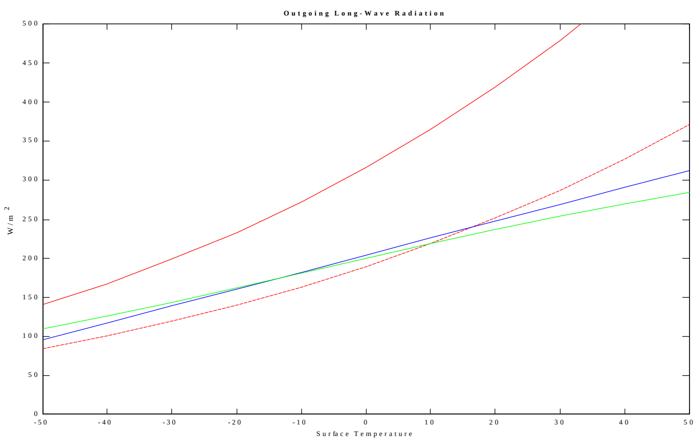
Solar Insolation, and the effect of obliquity

- ♦ Observations indicate that...
- ♦ S(sinθ) ≈ S_{bar}{1 - 0.477 P_2(sinθ)}
  - where $P_2(x) = (3x^2 - 1) / 2$
    - (second Legendre polynomial)
- ♦ ∴ S(equ) = 1.239 × 340 = 421 W m^{-2}
    - (c.f. 433 W m^{-2} ignoring obliquity)
- ♦ S(pole) = 0.523 × 340 = 178 W m^{-2}
- ♦ ∴ the main effect of obliquity is polar warming)
    - N.B. with very high obliquity, the poles may be warmer than the equator !!!

Outgoing LW infra-red radiation

- Budyko's Linear Approximation
  - $F \approx 204 + 2.17\, T_s$ $\quad Wm^{-2}$
  - already (implicitly) incorporates the greenhouse effect due to water vapour
  - good approximation to data for $0 < T_s < 30$ ºC
  - but may be poor if extrapolated outside that range
- or logistic form
  - $F \approx 0.9 \times \sigma\, (T_s+273)^4 \times [1\text{-logistic}\{(T_s - 50)/100\}]$
  - allows for saturation of F (at a maximum of ca 320 Wm^{-2}) due to absorption by water vapour

Outgoing Long-Wave Radiation and the Water Vapour Greenhouse Effect



Atmospheric& Oceanic heat transport

- of both latent & sensible heat
  - by advection (mean flow)
  - and by mixing (eddies)
  - due to baroclinic instability
  - parameterise as Fickian diffusion : $Q = K\, \partial T/\partial y$
    - depends on local gradient, with K = constant, or variable
    - N.B : Stone's formulation (for atmospheric eddy mixing)

      $K\ \mu\ k\ (\partial T/\partial y)^n$ where 0.5 < n < 3 (varies with latitude)
- or can use Budyko's parameterisation
  - to obtain a point-wise local formulation
  - like "Newtonian cooling" : $Q = K' (T - T_{bar})$
  - useful for "sketching" the system (but peculiar...)

Heat Balance Equation

Evolution of temperature field (1D)

$$C \frac{\partial T_s(q)}{\partial t} = A(q)\left[(1-a)S(q) - F(T_s)\right] - \nabla . Q$$

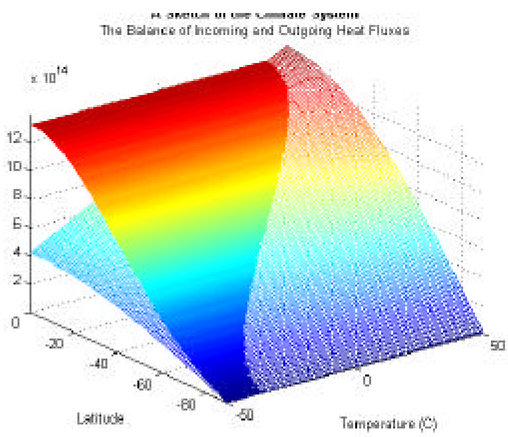
where Q=advective/diffusive heat flux

Sketching the System

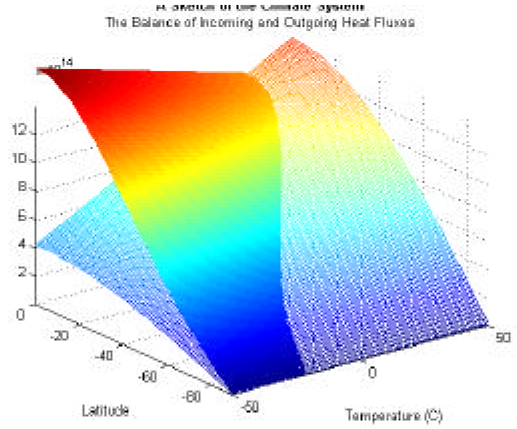
- Consider only the steady-state
- Use Budyko's formulation for mixing
- ∴ local balance for each latitude band
- with North's approximation for S
- and either Budyko or logistic formulation for F, i.e....

$$0.9 \times \sigma T_s^{\ 4} \times [1\text{-logistic}\{(T_s\text{-}50)/100\}]$$
$$\approx S_{bar}\{1 - 0.477\ P_2(\sin\theta)\} - K'\ (T_s - T_{bar})$$

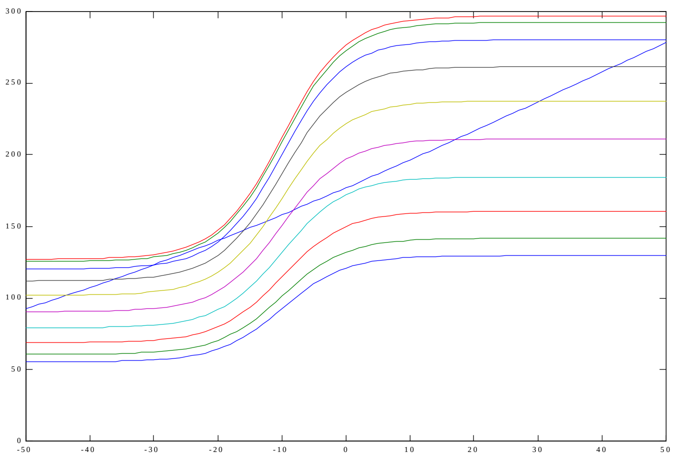
Without Ice Albedo Effect : No Atmospheric/Ocean Mixing



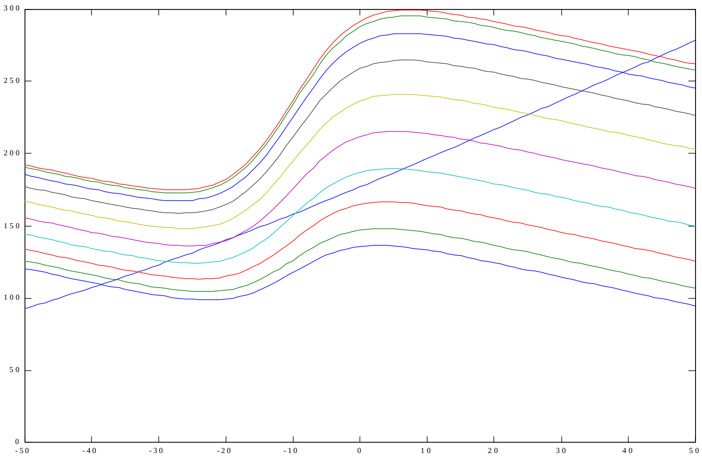
Without Ice Albedo Effect : Mixing = 2e3 mks



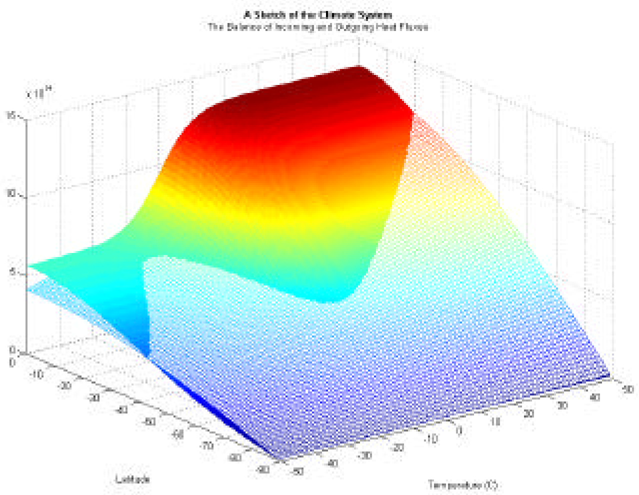
With Ice/Albedo and Water Vapour Feedbacks : Diffusivity =0



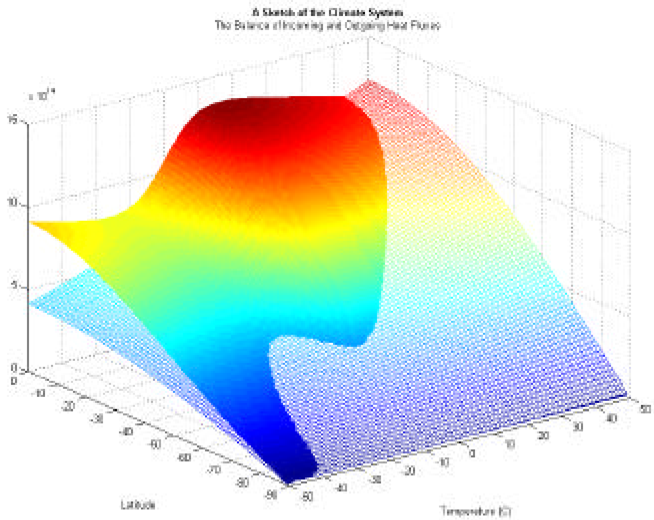
With Ice/Albedo and Water Vapour Feedbacks and Budyko-type mixing (illustrative)



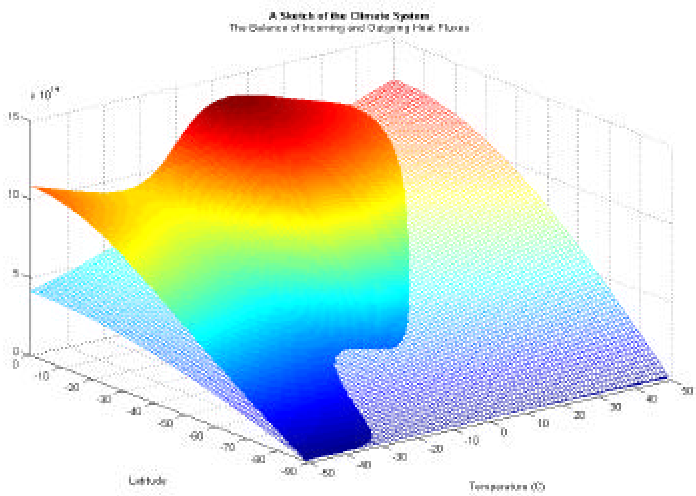
With Ice/Albedo and Water Vapour Feedbacks :Mixing =0



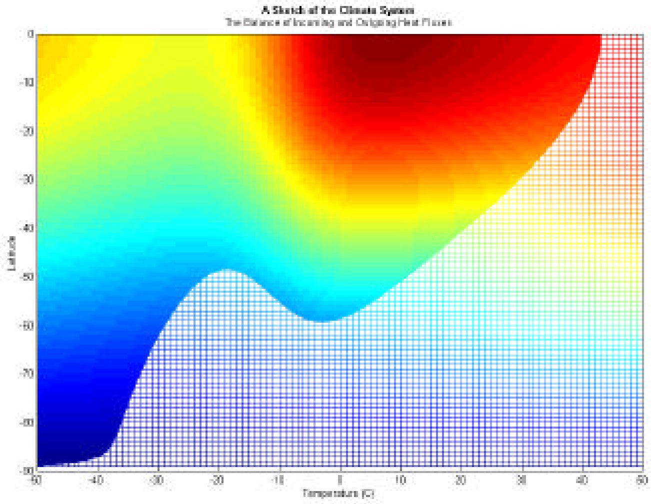
With Ice/Albedo and Water Vapour Feedbacks : mixing = 2E3 mks



With Ice/Albedo and Water Vapour Feedbacks : Diffusivity = 3E3 mks



With Ice/Albedo and Water Vapour Feedbacks : Diffusivity = 2E3 mks



Incorporating proper (Fickian) mixing

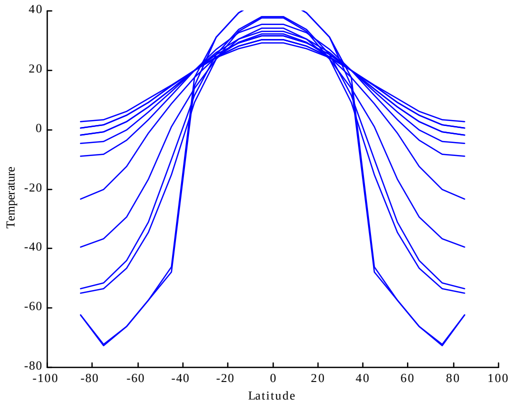
- Several methods are available...

  (a) Analytical solution using expansion in Legendre polynomials in sinθ (North 1975)

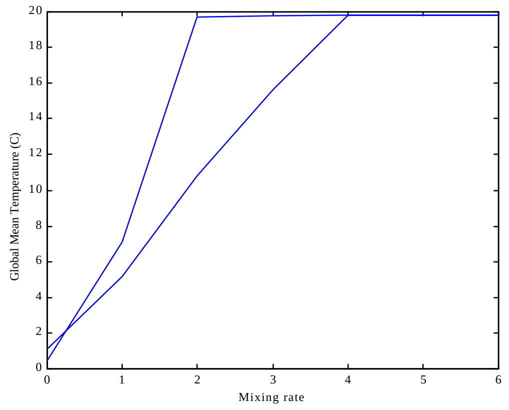
  (b) Numerical solutions

  - steady-state : iterative solution of linear equations
    - feasible using simple iteration with spreadsheets (see eg Excel spreadsheet provided by McG & H-S)
    - or matrix methods
  - time-dependent : all usual methods for first-order ordinary or partial differential equations (explicit, implicit, etc)

EBM with Ice-Albedo effect : no flow : mixing 0 to 6 e4 m^2s^{-1}



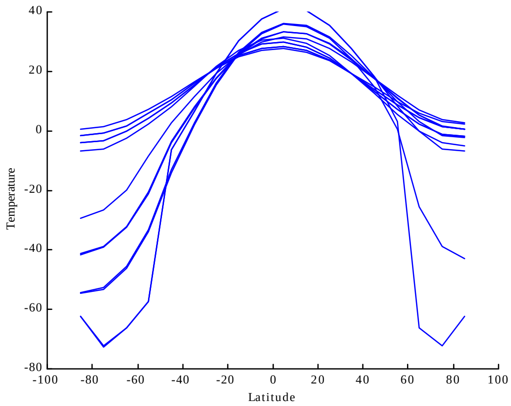
EBM with Ice-Albedo effect : no flow : mixing 0 to 6 e4 m^2s^{-1}



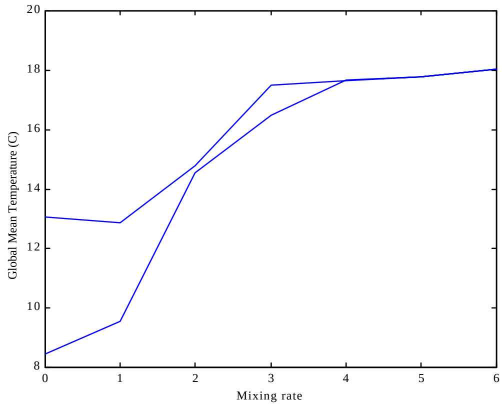
Adding explicit oceanic transport

- Assume same temperature for atmosphere & surface ocean (tight coupling = rapid exchange)
- Diffusive treatment for mixing (in both)
  - actual location is irrelevant
- Advective transport (overturning) in ocean
  - introduces asymmetry between N & S hemispheres
  - may be modelled (THC) or specified (stream function)
  - first approximation : single box for deep ocean

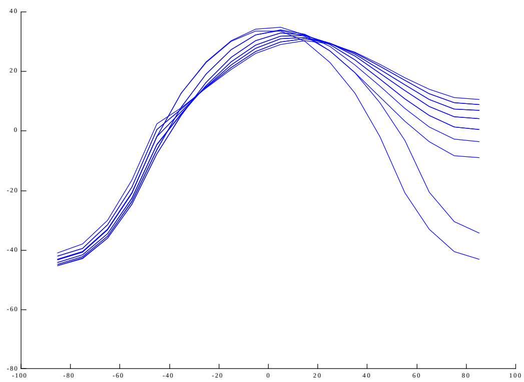
EBM with Ice-Albedo effect : flow=20Sv : mixing 0 to 6 e4 m^2s^{-1}



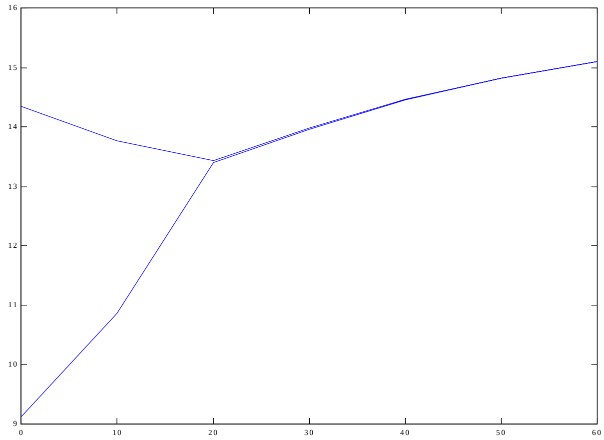
EBM with Ice-Albedo effect : flow=20Sv : mixing 0 to 6 e4 m^{2}s^{-1}



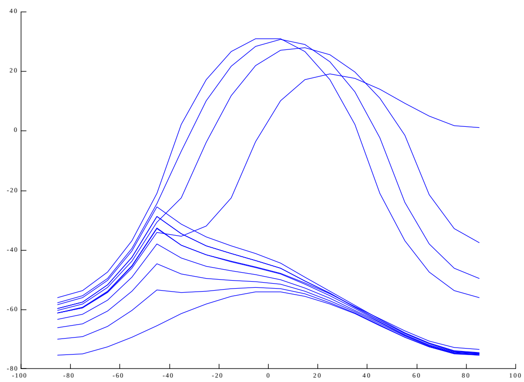
Meridional temperature distribution for varying THC rates (0 to 60 Sv and back down)
: with ice/albedo and water vapour feedbacks, K=2E4 m^2/sec



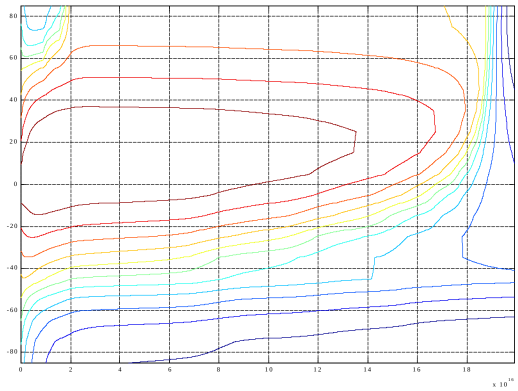
Global mean temperature for varying THC rates (0 to 60 Sv and back down)
: with ice/albedo and water vapour feedbacks, K=2E4 m^2/sec



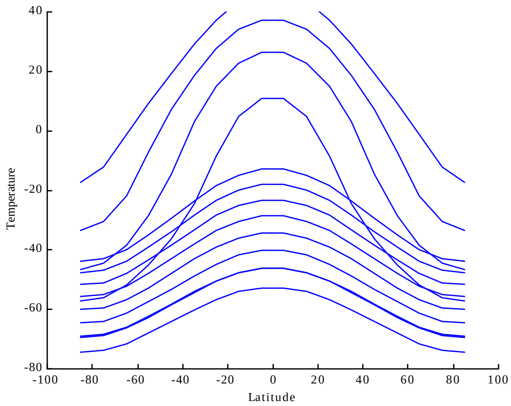
Meridional temperature distribution for varying THC rates (0 to 60 Sv and back down)
: with ice/albedo and water vapour feedbacks, K=2E4 m^2/sec
Ice albedo =0.75



Longer-term evolution of temperature distribution : THC = 30 Sv
(with ice/albedo and water vapour feedbacks, K=2E4 m^2/sec : ice albedo =0.75)



EBM with Ice-Albedo effect : flow=0 : mixing 2 e4 m^2s^{-1}
varying solar constant



EBM with Ice-Albedo effect : flow=0 : mixing 2 e4 m^2s^{-1}
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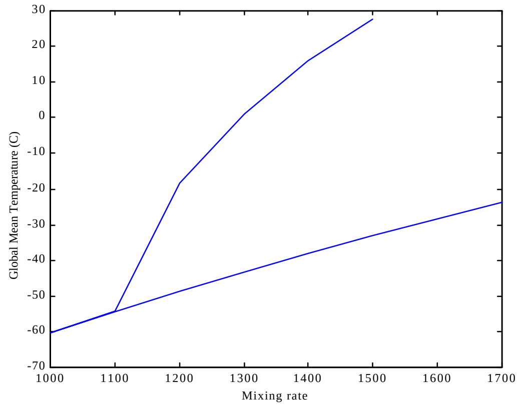

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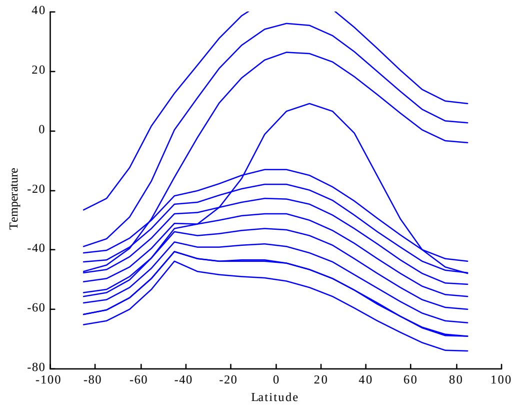

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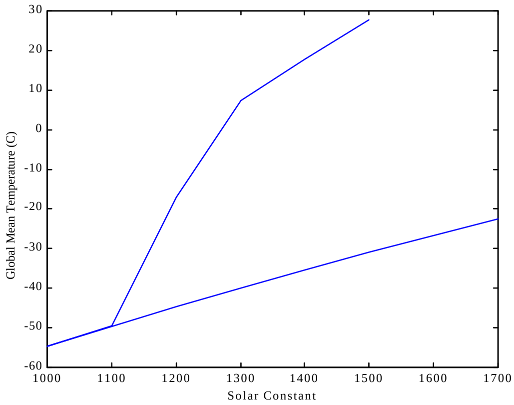

Forcing the system
(Climate sensitivities)

- ♦ Values of λ (K per W m^{-2})
  - black body 0.30
  - + water vapour (g-h effect) 0.46
  - + ice-albedo feedback 0.64 (polar ice only)
  - + fixed ocean advection no change
  - if → snowball state 0.24 (colder)
  -
- ♦ Doubling CO2 ⇒ forcing ≈ +4 W m^{-2}
  - ⇒ **ΔT** ≈ 2.5 ºC